Exercice - M0085C

Donnons la valeur exacte de chacun des nombres suivants :

$$\cos\left(\frac{5}{4}\right) \quad \cos\left(\frac{11\pi}{6}\right) \quad \cos\left(\frac{11\pi}{3}\right) \quad \cos\left(-\frac{3\pi}{4}\right) \quad \cos\left(\frac{2011\pi}{2}\right)$$

$$\sin\left(-\frac{\pi}{4}\right) \quad \sin\left(\frac{13\pi}{6}\right) \quad \sin\left(-\frac{10\pi}{3}\right) \quad \sin\left(\frac{76\pi}{3}\right) \quad \sin\left(\frac{2011\pi}{4}\right)$$

Calculons les cosinus de la première série de nombres.

$$\cos\left(\frac{5\pi}{4}\right) = \cos\left(\pi + \frac{\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{11\pi}{6}\right) = \cos\left(\frac{11\pi}{6} - 2\pi\right) = \cos\left(-\frac{11\pi}{6}\right) = \cos\left(-\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{11\pi}{3}\right) = \cos\left(\frac{11\pi}{3} - 4\pi\right) = \cos\left(-\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$\cos\left(-\frac{3\pi}{4}\right) = \cos\left(\frac{\pi}{4} - \pi\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{2011\pi}{2}\right) = \cos\left(\frac{(4 \times 502 + 3)\pi}{2}\right) = \cos\left(502 \times 2\pi + \frac{3\pi}{2}\right)\cos\left(\frac{3\pi}{2}\right) = 0$$

Calculons les sinus de la deuxième série de nombres.

$$\sin\left(-\frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

$$\sin\left(\frac{13\pi}{6}\right) = \sin\left(2\pi + \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\sin\left(-\frac{10\pi}{3}\right) = \sin\left(-\frac{\pi}{3} - \pi\right) = -\sin\left(\frac{\pi}{3} + \pi\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{76\pi}{3}\right) = \sin\left(\frac{(6 \times 12 + 4)\pi}{3}\right) = \sin\left(12 \times 2\pi + \frac{4\pi}{3}\right) = \sin\left(\frac{4\pi}{3}\right) = \sin\left(\frac{\pi}{3} + \pi\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{2011\pi}{4}\right) = \sin\left(\frac{(8 \times 251 + 3)\pi}{4}\right) = \sin\left(251 \times 2\pi\frac{3\pi}{4}\right) = \sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$$