Exercice - M0087C

1) Résolvons l'équation : $2\cos(2x)+\sqrt{3}=0$

$$\begin{aligned}
&2\cos(2x)+\sqrt{3}=0\\
\Longleftrightarrow\ &2\cos(2x)=-\sqrt{3}\\
\Longleftrightarrow\ &cos(2x)=-\frac{\sqrt{3}}{2}\\
\Longleftrightarrow\ &cos(2x)=\cos\left(\frac{5\pi}{6}\right)\\
\Longleftrightarrow\ &2x=\frac{5\pi}{6}+2k\pi \quad k\in\mathbb{Z} \qquad \text{ou} \qquad 2x=-\frac{5\pi}{6}+2k\pi \quad k'\in\mathbb{Z}\\
\Longleftrightarrow\ &x=\frac{5\pi}{12}+k\pi \quad k\in\mathbb{Z} \qquad \text{ou} \qquad x=-\frac{5\pi}{12}+k'\pi \quad k\in\mathbb{Z}
\end{aligned}$$

En se limitant aux solutions de l'intervalle $]-\pi,+\pi[$

$$-\pi<\frac{5\pi}{12}+k\pi<\pi \qquad \text{ou} \qquad -\pi<-\frac{5\pi}{12}+k'\pi<\pi$$

$$-\pi-\frac{5\pi}{12}<k\pi<\pi-\frac{5\pi}{12} \qquad \text{ou} \qquad -\pi+\frac{5\pi}{12}<k'\pi<\pi+\frac{5\pi}{12}$$

$$-\frac{17\pi}{12}<k\pi<\frac{7\pi}{12} \qquad \text{ou} \qquad -\frac{7\pi}{12}<k'\pi<\frac{17\pi}{12}$$

$$-\frac{17}{12}<k<\frac{7}{12} \qquad \text{ou} \qquad -\frac{7}{12}<k'<\frac{17}{12}$$

En résumé

$$k\in\{-1,0\} \qquad \text{ou} \qquad k'\in\{0,1\}$$

Conclusion :

$$S=\left\{-\frac{7\pi}{12},-\frac{5\pi}{12},\frac{5\pi}{12},\frac{7\pi}{12}\right\}$$

2) Résolvons l'équation : $\sqrt{2}\sin\left(4x-\frac{\pi}{6}\right)=1$

$$\begin{aligned}
&\sqrt{2}\sin\left(4x-\frac{\pi}{6}\right)=1\\
\Longleftrightarrow\ &\sin\left(4x-\frac{\pi}{6}\right)=\frac{1}{\sqrt{2}}\\
\Longleftrightarrow\ &\sin\left(4x-\frac{\pi}{6}\right)=\frac{\sqrt{2}}{2}\\
\Longleftrightarrow\ &\sin\left(4x-\frac{\pi}{6}\right)=\sin\left(\frac{\pi}{4}\right)\\
\Longleftrightarrow\ &4x-\frac{\pi}{6}=\frac{\pi}{4}+2k\pi \quad k\in\mathbb{Z} \qquad \text{ou} \qquad 4x-\frac{\pi}{6}=\pi-\frac{\pi}{4}+2k'\pi \quad k'\in\mathbb{Z}\\
\Longleftrightarrow\ &4x=\frac{\pi}{6}+\frac{\pi}{4}+2k\pi \quad k\in\mathbb{Z} \qquad \text{ou} \qquad 4x=\frac{\pi}{6}+\frac{3\pi}{4}+2k'\pi \quad k'\in\mathbb{Z}\\
\Longleftrightarrow\ &4x=\frac{5\pi}{12}+2k\pi \quad k\in\mathbb{Z} \qquad \text{ou} \qquad 4x=\frac{11\pi}{12}+2k'\pi \quad k'\in\mathbb{Z}\\
\Longleftrightarrow\ &x=\frac{5\pi}{48}+k\frac{\pi}{2} \quad k\in\mathbb{Z} \qquad \text{ou} \qquad x=\frac{11\pi}{48}+k'\frac{\pi}{2} \quad k'\in\mathbb{Z}
\end{aligned}$$

En se limitant aux solutions de l'intervalle $]-\pi,+\pi[$

$$-\pi<\frac{5\pi}{48}+k\frac{\pi}{2}<\pi \qquad \text{ou} \qquad -\pi<\frac{11\pi}{48}+k'\frac{\pi}{2}<\pi$$

$$-\frac{53\pi}{48}<k\frac{\pi}{2}<\frac{43\pi}{48} \qquad \text{ou} \qquad -\frac{59\pi}{48}<k'\frac{\pi}{2}<\frac{37\pi}{48}$$

$$-\frac{53}{24} < k < \frac{43}{24} \qquad \text{ou} \qquad -\frac{59}{24} < k' < \frac{37}{24}$$

En résumé

$$k \in \{-2, -1, 0, 1\} \qquad \text{ou} \qquad k' \in \{-2, -1, 0, 1\}$$

Conclusion :

$$S = \left\{ -\frac{43\pi}{48}, -\frac{37\pi}{48}, -\frac{19\pi}{48}, -\frac{13\pi}{48}, \frac{5\pi}{48}, \frac{11\pi}{48}, \frac{29\pi}{48}, \frac{35\pi}{48} \right\}$$