Exercice - M0139C

Nous avons :

$$z_1 = 2 + i \qquad z_2 = -1 + 3i \qquad z_3 = 5 - 2i$$

1) Calculons $z_1 + z_2$

$$z_1 + z_2 = 2 + i + (-1 + 3i) = 1 + 4i$$

Conclusion :

$$z_1 + z_2 = 1 + 4i$$

2) Calculons $3z_1 - 2z_3$

$$3z_1 - 2z_3 = 3(2 + i) - 2(5 - 2i) = 6 + 3i - 10 + 4i = -4 + 7i$$

Conclusion :

$$3z_1 - 2z_3 = -4 + 7i$$

3) Calculons $z_1 \times z_2$

$$z_1 \times z_2 = (2 + i)(-1 + 3i) = -2 + 6i - i + 3i^2 = -2 - 3 + 5i = -5 + 5i$$

Conclusion :

$$z_1 \times z_2 = -5 + 5i$$

4) Calculons $z_2 \times z_3$

$$z_2 \times z_3 = (-1 + 3i)(5 - 2i) = -5 + 2i + 15i - 6i^2 = -5 + 6 + 17i = 1 + 17i$$

Conclusion :

$$z_2 \times z_3 = 1 + 17i$$

5) Calculons $\dfrac{5}{z_2}$

$$\frac{5}{z_2} = \frac{5}{-1 + 3i} = \frac{5(-1 - 3i)}{(-1 + 3i)(-1 - 3i)} = \frac{-5 - 15i}{(-1)^2 + (-3)^2} = \frac{-5 - 15i}{10}$$

Conclusion :

$$\frac{5}{z_2} = -\frac{1}{2} - i\frac{3}{2}$$

6) Calculons $\dfrac{z_2}{z_1}$

$$\frac{z_2}{z_1} = \frac{-1 + 3i}{2 + i} = \frac{(-1 + 3i)(2 - i)}{(2 + i)(2 - i)} = \frac{-2 + i + 6i + 3}{2^2 + 1^2} = \frac{1 + 7i}{5}$$

Conclusion :

$$\frac{z_2}{z_1} = \frac{1}{5} + i\frac{7}{5}$$

7) Calculer z_3^2

$$z_3^2 = (5 - 2i)^2 = 5^5 - 2 \times 5 \times 2i + (2i)^2 = 25 - 20i - 4 = 21 - 20i$$

Conclusion :

$$z_3^2 = 21 - 20i$$